Turbulence-Induced Spectral Bias in Laser Anemometry

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Simulation studies are performed to investigate various spectral estimators used in laser anemometry. Autoregressive models are used to generate data in which the distribution of particle arrival times is correlated with the instantaneous, measured velocity component, as is encountered in highly turbulent flows. The influence of particle seed rate, turbulence level, and processing parameters on the spectral estimation is studied systematically for three commonly used spectral estimators. Significant distortion of the spectrum is found for turbulence levels above approximately 30%.

I. Introduction

THE task of generating spectral estimates from data obtained using a laser anemometer has been treated extensively in recent years by numerous authors,[1-3] resulting in a number of different approaches with varying degrees of computational efficiency and statistical variability. In general, however, the data are assumed to be randomly sampled, i.e., the distribution of the particle arrival times in the measuring control volume is assumed (or prescribed in simulation studies) to be Poisson. Although this is a good approximation for low turbulence levels, it is not appropriate for high turbulence levels as would be encountered, for instance, in separated or recirculating flows. The distribution of particle arrival times is, in general, given by a Poisson process with a random rate parameter, which is correlated with the instantaneous flow velocity. This causes a deviation from the Poisson distribution, especially at high turbulence levels, and is responsible for the so-called velocity bias of the ensemble mean and variance estimators as investigated previously by numerous authors.[4,5] A bias of the ensemble variance will, out of necessity, also affect any spectral estimator based on all available data points, and it is the distortion of the resulting spectrum that is the subject of the present paper.

The effect of the particle arrival statistics on spectral estimates from laser-anemometer data has not yet been investigated directly, although most authors are aware of the potential distortion of the spectrum. For example, Roberts et al.[6] first established that the probability density function (pdf) of the arrival times in their experiment was exponential before proceeding with computations of the spectrum. This would correspond to a position in the flow with a low turbulence level. Buchhave et al.[2] provide a means of eliminating any effect of the particle statistics by considering also the resident times of the individual particles. This approach is, however, only applicable if the resident times are available and if the so-called slot-correlation technique is employed. Using the direct-transform approach, there is no known means of treating this biasing effect, although Roberts and Gaster[3] have made some speculations about possible approaches.

In order to study this effect systematically, laser-anemometry data with known moments and spectral content have been generated numerically using autoregressive models of the first and second order. Details of the simulation process and verifications of the signal properties are given in Sec. II. These data are then processed using the direct-transform technique commonly used for generating spectral estimates and described in detail by Roberts et al.[6] In addition, two further processing methods are implemented. The first is the case of a "controlled processor," i.e., a processor that cannot, or is not allowed to sample every available data point. The second variation is the commonly used approach of resampling, at regular intervals, the analog output of the signal processor which uses a "hold" circuit during dropout periods. Further reviews of these techniques are available in previous publications, e.g., Bell[7] and Srikantaiah and Coleman.[8] A comparison of the various approaches and the influence of the particle seed rate and the turbulence level is presented in Sec. III.

II. Signal Processing

A. Data Generation

To simulate LDA data properly, a doubly stochastic Poisson process must be generated, as discussed above. To do this, an autoregressive model of order two has been employed to generate first a time series at regular, and very closely spaced points in time, which is hereafter referred to as the primary time series. This near-continuous signal is then seeded with particles to yield a secondary time series representing the simulated LDA data. The autoregressive model used takes the form

$$z_k = \phi_1 z_{k-1} + \phi_2 z_{k-2} + a_k$$
$$\phi_2 + \phi_1 < 1, \qquad \phi_2 - \phi_1 < 1, \qquad -1 < \phi_2 < 1, \tag{1}$$

where the kth sample z_k is related to the two previous samples z_{k-1}, z_{k-2} through the coefficients ϕ_1 and ϕ_2, and to a Gaussian distributed random number a_k. A mean value μ_z can be added to each sample subsequent to the generation according to Eq. (1). The resultant time series has energy up to a frequency of 0.5, the spectrum given by

$$S(f) = \frac{2\sigma_a^2}{1 + \phi_1^2 + \phi_2^2 - 2\phi_1(1-\phi_2)\cos 2\pi f - 2\phi_2 \cos 4\pi f} \qquad 0 \le f \le 0.5 \tag{2}$$

with σ_a^2 being the variance of the random numbers used. The primary time series will also be Gaussian distributed, and its

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variance is related to σ_a^2 through

$$\sigma_z^2 = \frac{(1-\phi_2)}{(1+\phi_2)} \frac{\sigma_a^2}{(1-\phi_2)^2 - \phi_1^2} \tag{3}$$

The turbulence intensity, given by $\eta = \sigma_z/\mu_z$, can therefore be adjusted between 0 and ∞ through the choice of σ_z and/or μ_z. In fact, by adjusting μ_z alone, the same time series could be used for all desired values of η. The correlation coefficient of the process is given recursively as

$$\rho_0 = 1, \quad \rho_1 = \phi_1/(1-\phi_2), \quad \rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} \tag{4}$$

and thus the integral time scale θ_c of the signal fluctuations can be computed by integrating ρ. For a first-order process, i.e., $\phi_2 = 0$, the integral time scale reduces to

$$\theta_c = 1/(1-\phi_1) \tag{5}$$

Details of autoregressive models can be found in Box and Jenkins.[9]

For generating data samples with either Poisson-distributed arrival times or velocity-correlated arrival times, it is necessary first to determine the particle arrival time and then to interpolate the velocity at that instant in time, which will fall somewhere between two of the samples in the primary time series. By choosing the coefficient ϕ_1 and ϕ_2 such that the integral time scale θ_c is larger than the time step used in the primary series, the interpolation has little effect on the spectrum below $f = 0.5$, as confirmed by preliminary investigations. At this point it should be noted that all further time and velocity quantities are assumed to be expressed relative to the time step used in generating the primary series, which was arbitrarily set to 1. Thus, dimensions are no longer explicitly given.

The computation of particle arrival times is trivial for the case of a Poisson distribution, since the interarrival times themselves will be exponentially distributed. Therefore once a mean rate ν is prescribed, individual arrival times can be generated using

$$t_n = -(1/\nu)\ln(1-a_n) \tag{6}$$

where a_n are random numbers uniformly distributed between 0 and 1. The particle rate is expressed in nondimensional form as

$$r = \nu\theta_c \tag{7}$$

the reduced particle seed rate, which is the mean number of particles per integral time scale.

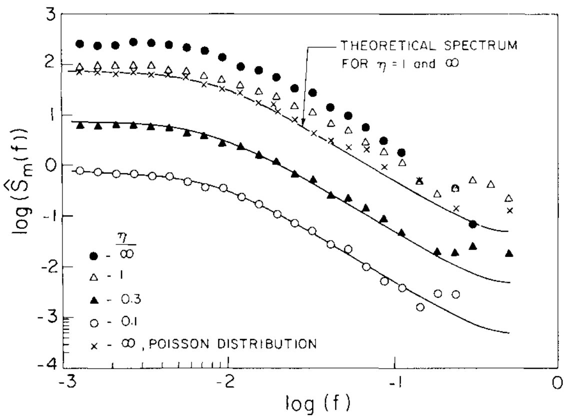

Fig. 1 Computed spectra using the direct-transform method for various turbulence levels ($\phi_1 = 0.95$, $\phi_2 = 0$, $r = 10$, $T_B = 50\ \theta_c$).

When the arrival times are to be correlated with the velocity, the distance between particles x_n, rather than the time between particles, is prescribed, and using the one-dimensional conveyor-belt model, the arrival time t_n is computed by solving the integral equation

$$x_n = \int_{t_{n-1}}^{t_n} v(t)\,\mathrm{d}t \tag{8}$$

Again, since θ_c was chosen to be large, a simple trapezoidal integration was adequate to solve Eq. (8). The interparticle distances were generated using Eq. (6), replacing t_n with x_n. The particle rate ν is replaced with A, the number of particles per unit length. Therefore, the mean nondimensional particle rate becomes

$$r = Av_0\theta_c \tag{9}$$

with v_0 being the mean of the absolute velocity. For low turbulence levels, i.e., for $\sigma_z/\mu_z \ll 1$, v_0 corresponds to the mean of the autoregressive model μ_z. For higher turbulence levels, however, significant reverse flow occurs, and v_0 must be computed by taking the first moment of the Gaussian pdf with respect to $|z|$. This was done using Hermite integration. Once r was prescribed, A could then be obtained from Eq. (9) for use in generating the x_n. Two programs were available for generating either Poisson-distributed sample times or velocity-correlated sample times. In either case, the secondary time series was stored on disk with alternating values of velocity and arrival times, as would be expected from an LDA counter processor. The primary time series was never stored.

B. Processing Algorithms

The first of the three processing algorithms used was the direct-transform method as described in Ref. 5:

$$S(f_k) = \frac{2}{\nu^2 T}\left[\left|\sum_j x(t_j)D(t_j)e^{-i2\pi f_k t_j}\right|^2 - \sum_j x^2(t_j)D^2(t_j)\right] \tag{10}$$

The spectral estimates were computed at n_f logarithmically spaced frequency values f_k between a given minimum value and 0.5. The total available record was divided into N_B short blocks, each of duration T_B, and spectral estimates were computed according to Eq. (10) for each block. The final spectrum was obtained by averaging at each frequency f_k over all blocks, a method that reduces the variability of the estimates in proportion to the number of blocks.[3] The variability was reduced further by removal of the block mean as outlined by Roberts et al.[6] For final scaling of the spectrum, the mean particle rate ν was determined simultaneously with the processing. Because of the large number of data used for processing, no additional windowing was used, i.e., $D(t) = 1$. The spectral estimate computed in this manner is designated as $S_m(f_k)$.

The second processing method is identical to the direct-transform method, except that it operates on a subset of the available data as seen by a "controlled" processor. A controlled processor is one that is not allowed to, or is not capable of processing each Doppler burst. This occurs when requested data transfer rates exceed the interface specifications, or when the processor itself is not extremely fast (e.g., processing by means of a transient recorder and a computer). Purposely inhibiting an otherwise fast or "free-running" processor to become a controlled processor also has been shown to be an effective means of reducing the turbulence-induced statistical bias in certain cases, as outlined by Erdmann and Tropea.[4] For this approach a time interval is prescribed at which the processor is regularly enabled. The first sample following the enabling is taken into the data set to be processed. All others up to the next enabling event are discarded. The arrival times of the sampled data are assumed, however, to be accurate, i.e., the interarrival times of the discarded data are added to that of the sampled point. The enabling interval T_e is expressed nondimensionally using the integral time scale, i.e.,

$$q = T_e/\theta_c \tag{11}$$

and is known as the reduced control period.

The final approach investigated is of practical significance, since it is perhaps the most widely used method to date. In this approach the analog output of the Doppler processor is resampled at regular intervals using an analog-to-digital converter. Spectral estimates are computed using conventional fast Fourier transform (FFT) processing algorithms (see, e.g., Bendat and Piersol[10]). In generating an analog signal, most commercial Doppler processors (e.g., Tracker, Counter) hold the voltage constant at a value proportional to the last validated burst until the next burst has been validated. In implementing this scheme with the present simulation algorithms, only the sample interval had to be additionally prescribed. Again, this was expressed with respect to the integral time scale of the primary time series, i.e.,

$$s = \Delta t/\theta_c \tag{12}$$

As with the direct-transform approach, the total available record was subdivided into blocks, over which averaging of the spectra were performed. The number of samples in each individual block was kept to a power of 2. The frequency resolution was then given by the inverse of the block length, and the maximum frequency resolved was $1/2\Delta t$. This spectral estimate is designated $\hat{S}_a(f)$.

III. Results

The results presented here have been obtained using a first-order autoregressive model with the coefficients $\phi_1 = 0.95$ and $\phi_2 = 0$, however similar conclusions were reached for the second-order models used. The integral time scale of this process was thus $\theta_c = 20$. In all cases, processing was performed over 10^5 samples as suggested by Srikantaiah and Coleman.[8] In performing block averages, the average number of samples in a block, n_B, was kept above a minimum of 50 examples, as recommended in Ref. 3, and usually many more were used, depending on the particle density.

A. Processing with the Direct Transform Method

Figure 1 shows spectra for $\eta = 0.1$, 0.3, 1, and ∞, as computed according to Eq. (10), keeping other parameters constant ($r = 10$, $T_B = 50\,\theta_c$). First, it can be seen that the variance of the signal, i.e., the area under the spectrum, is overestimated at high turbulence levels, as predicted in Ref. 4. Figure 1, however, also indicates that in absolute terms the lower-frequency components are more directly affected by the bias. Indeed, higher-frequency fluctuations will, in general, be of smaller amplitude and thus of smaller particle rate differential than low-frequency fluctuations. Thus, in an absolute sense, the bias will not be felt as strongly at higher frequencies. Similar results, which are not presented here, have been obtained for particle densities of $r = 1$ and 100. In agreement with these observations, the bias of the variance has been predicted by Buchhave[11] and Erdmann and Tropea[4] to be independent of particle rate for a free-running processor, i.e., a processor that uses all particles.

Included in Fig. 1 for comparison is a computation of the spectrum using a direct-transform method from a secondary time series that was generated with Poisson arrival times. This spectrum is completely void from bias effects, even for the case of infinite turbulence intensity.

B. Processing with a Controlled Processor

It has been shown that the effects of the velocity-correlated particle statistics on the first two moments of the velocity can be reduced significantly using a controlled processor for particle densities above approximately $r = 1$, the exact value depending on the reduced control period q^4. For a value of $\eta = 1$ and $r = 10$, for example, a controlled processor operating with $q = 1$ would eliminate the normalized bias of the mean velocity to less than 5% of its normal expected value of η^2. Therefore, the spectra should also show less distortion for similar values of q.

Figure 2 illustrates spectra computed using the direct-transform method but on the reduced subset of data seen by a controlled processor with various q values. The results for the free-running processor ($q = 0.0$) can be compared directly to the results in the previous figure. The spectrum is well estimated in the low-frequency range for larger values of q, apparently eliminating distortion due to bias. Two additional effects can, however, be recognized from this figure. First, the number of available data points per block, η_B (or, more precisely, the mean effective data rates) decreases with increasing q, thus contributing to a higher variability. Second, in contrast to simply reducing the particle rate through the parameter r, the controlled processor selectively reduces the particle rate such that small interarrival times are no longer included. Thus, the accuracy of the spectral estimates at high frequencies decreases disproportionately to the overall increase in variability. In fact, in Fig. 2 many of the estimates at higher frequencies had to be discarded because they were negative. Thus, for very large values of q (>1), the accuracy of the estimates again decreases. At lower particle densities, this behavior remains virtually unchanged.

Thus, although the controlled processor presents a viable means of controlling the bias of the ensemble mean velocity, its use is much more restrictive for the measurement of power spectral densities.

C. Processing Using an Analog Output Signal

Whereas the direct transform offers the possibility of alias-free spectral estimates, the resampling of the analog output of the Doppler processor offers significant speed advantages (through the use of the FFT) and removes the necessity of measuring particle arrival times.

At high particle rates the analog signal becomes nearly continuous, and this approach is expected to perform well at all levels of turbulence. Figure 3 shows a spectrum for the case of $\eta = \infty$ and indicates that this is true even for the modest particle density of $r = 10$. A comparison with the direct transform presented in Fig. 1 using the same second-

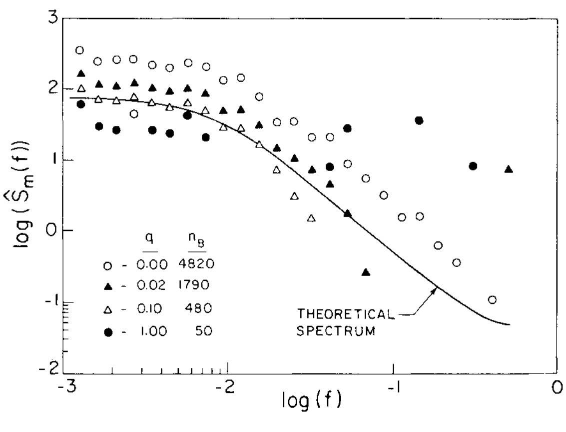

Fig. 2 Spectra computed using the direct-transform method with a controlled processor ($\phi_1 = 0.95$, $\phi_2 = 0$, $\eta = \infty$, $r = 100$).

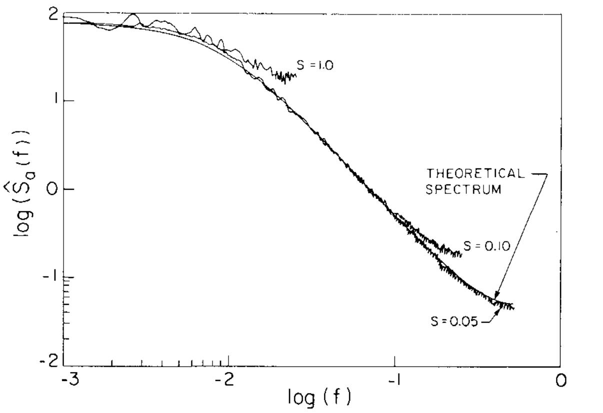

Fig. 3 Spectrum computed using a simulated analog output ($\phi_1 = 0.95$, $\phi_2 = 0$, $r = 10$).

ary time sequence shows that the spectral distortion due to the velocity-correlated arrival times has been entirely removed.

The maximum frequency resolved using this method is given by $1/2\Delta t$, or $1/(2\times s\times\theta_c)$, as seen in Fig. 3. Regardless of the value of s, however, an increase of the spectral estimates is seen near the maximum frequency. This is observed at all particle rates and is probably an aliasing effect to which this method is susceptible. The steplike nature of the reconstructed analog signal would introduce higher-frequency components, which are shown folded back here. In practical systems, an appropriately selected low-pass filter would eliminate these effects.

At very low particle rates, this method of processing is not expected to perform as well, and further investigations have indicated that this is already the case at $r = 1$. The spectrum is underpredicted at high frequencies and overpredicted at low frequencies, although not by as much as the bias using the direct transform. Whereas the bias error decreases with decreasing η, this spectral estimator does not improve at low particle rates. In fact, the same estimator was used to process data generated with Poisson distribution of particle arrival times, and no significant difference in the spectral estimates could be detected. Thus, the distortion at low particle rates using the resampling approach is related solely to the low particle rate and not to the particle arrival statistics inherent in laser anemometry. This conclusion is fully corroborated by theoretical work published in a recent paper by Adrian and Yao[12] and is termed step noise. Several other conclusions of the present work have also been theoretically derived.

IV. Discussion and Conclusions

The one major assumption involved in producing the preceding results is that the flowfield is strictly one-dimensional. The quantitative effect of this assumption is difficult to assess; however, qualitatively it can be said that the one-dimensional turbulence on the component of velocity being measured by the LDA will produce a maximum of the bias, since the correlation between the measured velocity and the instantaneous particle arrival rate will be a maximum under these conditions. Therefore, although the magnitude of the distortion to the spectrum shown here will, in general, not apply to all cases, the conditions under which it exists will probably remain the same. Conversely, situations in which no bias was found will also show no bias, even in three-dimensional turbulence fields.

It can be concluded from this investigation that the direct-transform approach of generating spectral estimates from laser anemometer data produces reliable results for low turbulence levels ($\eta < 0.3$), even for low particle densities. For modest to high particle densities, and for the entire range of turbulence levels, it can, however, be more efficient computationally to choose the analog signal approach. It should be noted that the hardware for this method, i.e., the A/D converter, etc., is not necessarily required, since the resampling of the inherently digital data can be performed, as in the present study, directly from the arrival time and velocity data. In fact, it may be possible to reduce the observed aliasing effects by employing an interpolating polynominal between data points rather than simulating the dropout hold circuit of typical Doppler processors.

It is clear however, that the direct-transform approach does not do a good job of spectral estimation at high turbulence levels, although qualitatively the computed spectrum will be correct. For all of these conclusions, it is assumed that the computations are being performed with proper regard to the operating parameters, such as block length and number of averaging blocks, as discussed elsewhere.[8]

Spectral analysis of data received from a controlled processor does not appear to be a viable means of reducing distortion due to particle arrival statistics, except for very special cases. This is a contrast to the fairly useful role the controlled processor plays in reducing the velocity bias of the mean and variance estimators.

The most difficult operating conditions appear to be the case of low particle densities ($r < 1$) and high turbulence levels ($\eta > 1$). No adequate means of spectral estimation was found for this case, and the choice of available options must be made accepting certain compromises in accuracy.

Acknowledgments

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